MATH 112 Review: Integration by parts

1. Find $\int x \cos x \, dx$.

Solution: Let $u = x$ and $dv = \cos x \, dx$. Then $du = dx$ and $v = \sin x$. Thus,

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C.$$

2. Find $\int \arctan x \, dx$.

Solution: Let $u = \arctan x$ and $dv = dx$. Then, $du = \frac{1}{x^2+1} dx$ and $v = x$. Thus,

$$\int \arctan x \, dx = x \arctan x - \int \frac{x}{x^2+1} \, dx = x \arctan x - \frac{1}{2} \ln\left(x^2 + 1\right) + C.$$

3. Find $\int e^x \sin x \, dx$.

Solution: Let $u = \sin x$ and $dv = e^x dx$. Then $du = \cos x \, dx$ and $v = e^x$. Thus,

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx.$$

Let $u = \cos x$ and $dv = e^x dx$. Then $du = -\sin x \, dx$ and $v = e^x$. Thus,

$$\int e^x \sin x \, dx = e^x \sin x - \left(e^x \cos x - \int e^x (-\sin x) \, dx \right) = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx.$$

If we set $I = \int e^x \sin x \, dx$, then the above becomes:

$$I = e^x \sin x - e^x \cos x - I.$$

Solving for I yields

$$I = (e^x \sin x - e^x \cos x)/2 + C.$$